Code No. 1571/CORE

FACULTY OF SCIENCE

M.Sc. IV- Semester Examination, October 2020

SUBJECT : Mathematics/Applied Maths / Maths with Computer Science

Paper – I : Integral Equations and Calculus of Variation

Time : 2 Hours **Max Marks : 80**

PART – A

Note: Answer any five questions. **(5x7=35 Marks)**

1. Define Resolvent kernel of volterra integral equation
2. Show that $\Gamma(n+1)=n!$ and $\beta(m, n) = \beta(n, m)$.
3. Solve the integral equation $\varphi(x) = \int_0^1 xt\varphi^2(t)dt$
4. Show that all iterated kernels of a symmetric kernel are symmetric
5. State and prove the fundamental Lemma of calculus of variations
6. Find the extremum of the functional $V[y(x)] = \int_0^{\pi/2} (y'^2 - y^2)\,dx,\ y(0) = 0,\ y(\pi/2) = 1$
7. Find the extremals of the functional $V[y(x)] = \int_0^{\pi/2} (y''^2 - y^2 + x^2)\,dx$
8. Derive the differential equation of a motion of simple pendulum using Lagrange's equation.

PART – B

Note: Answer any three questions. **(3x15=45 Marks)**

9 Transform the problem into integral equation $\frac{d^2y}{dx^2} + \lambda y = 0,\ y(0) = 1,\ y'(0) = 0$

10 Using the method of successive approximations, solve the integral equation

$$\varphi(x) = 1 + \int_0^x (x - t)\varphi(t)\,dt,\ \varphi_o(x) = 1$$

11 Solve the integro-differential equation.

$$\varphi''(x) + \int_0^x e^{2(x-t)}\varphi'(t)dt = e^{2x};\ \varphi(0) = 0, \varphi'(0) = 1$$

12 Solve the integral equation $\varphi(x) = 2\int_0^1 xt\ \varphi^3(t)\,dt$

13 On which curve the functional $\int_0^{\pi/2} [y'^2 - y^2 + 2xy]\,dy$ with y (0) = 0, $y(\pi/2) = 0$ be extremised.

14 Find the extremals of the functional

$$V[y(x), z(x)] = \int_0^{\pi/2} [y'^2 + z'^2 + 2yz]\,dx; \quad y(0) = 0,\ y(\pi/2) = 1,\ z(0) = 0,\ z(\pi/2) = -1.$$

15 State the Isoperimetric problem and find the extremals of the problem.

$V[y(x)] = \int_0^1 (y'^2 + x^2)dx$ given that $\int_0^1 y^2\,dx = 2,\ y(0) = 0,\ y(1) = 0$

16 Derive Lagrange equation of motion

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Code No. 1581/CORE

FACULTY OF SCIENCE

M.Sc. IV- Semester Examination, October 2020

SUBJECT : Applied Mathematics

Paper – III: Functional Analysis

Time : 2 Hours **Max. Marks: 80**

PART – A

Note : Answer any five questions. **(5x7=35 Marks)**

1. Prove that on a finite dimensional vector space, any two norms or equivalent
2. State and prove Translation invariance lemma
3. Prove that a linear functional f with domain D (f) in a normed space is continuous if and only if f is bounded
4. If in an inner product space $x_n \to x$ and $y_n \to y$, then prove that $< x_n, y_n > \to < x, y >$.
5. Let T be a bounded linear operator on a complex inner product space X and <Tx,x>=0 for all $x \varepsilon X$, then prove that T= O.
6. Prove that the product of two bounded self – adjoint linear operators A and B on a Hilbert space H is self – adjoint if and only if AB=BA.
7. Let X, Y be normed spaces and S, T εB (X,Y) Then prove t hat $(i)\ (S+T)^x = S^x + T^x$ (ii) $(\alpha T)^x = \alpha T^x$ for all scalar α.
8. Prove that the normed space X of all polynomials with norm defined by $\|x\| = \max_j \|\alpha_j\| (\alpha_0, \alpha_1, ...$ $thecoefficients\ of\ x)$ is not complete.

PART – B

Note : Answer any three questions. **(3x15=45 Marks)**

9. Prove that every finite dimensional subspace Y of a normed space X is complete and closed in X.

10 Let $T : D(T \to Y)$ be a bounded linear operator where D (T) lies in a normed space and Y is a Banach space. Then prove that T has an extension $\overline{D(T)} \to Y$ where $\overline{T}$ is a bounded linear operator of norm $\|\overline{T}\| = \|T\|$.

11 Prove that the vector space B (X,Y) of all bounded linear operators from a normed space X into a normed space Y is itself a normed space with norm defined by $\|T\| = \underset{\substack{x \in X \\ x \neq 0}}{SUP} \frac{\|Tx\|}{\|x\|}$.

Also prove that if Y is a Banach space, then B (X,Y) is a Banach space.

12 Let X be an inner product space and M a non – empty convex subset which is complete in the metric induced by the inner product. Then prove that for every given $x \varepsilon X$ there exists a unique $y \varepsilon M$ such that $\delta = \inf_{y \varepsilon M} \|x - y\| = \|x - y\|$.

13 Prove that an ortohonormal set M in a Hilbert space H is total if and only if for all $x \varepsilon H$ the Parsevel relation $\sum_k |\langle x, y \rangle|^2 = \|x\|^2$ holds.

14 State and prove Riesz's theorem for sesquilinear form.

15 State and prove Hahn-Banach theorem.

16 State and prove closed graph theorem.

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