FACULTY OF SCIENCE

M.Sc. IV- Semester Examination, October 2020

SUBJECT: Mathematics/Applied Maths / Maths with Computer Science

Paper - I: Integral Equations and Calculus of Variation

Time: 2 Hours

Max Marks: 80

PART - A

Note: Answer any five questions.

(5x7=35 Marks)

- 1. Define Resolvent kernel of volterra integral equation
- 2. Show that $\Gamma(n+1)=n!$ and $\beta(m, n)=\beta(n, m)$.
- 3. Solve the integral equation $\varphi(x) = \int_{0}^{1} xt \varphi^{2}(t)dt$
- 4. Show that all iterated kernels of a symmetric kernel are symmetric
- 5. State and prove the fundamental Lemma of calculus of variations
- 6. Find the extremum of the functional $V[y(x)] = \int_{0}^{\pi/2} (y'^2 y^2) dx$, y(0) = 0, $y(\pi/2) = 1$
- 7. Find the extremals of the functional $V[y(x)] = \int_{0}^{\pi/2} (y''^2 y^2 + x^2) dx$
- 8. Derive the differential equation of a motion of simple pendulum using Lagrange's equation.

PART - B

Note: Answer any three questions.

(3x15=45 Marks)

- 9 Transform the problem into integral equation $\frac{d^2y}{dx^2} + \lambda y = 0$, y(0) = 1, y'(0) = 0
- 10 Using the method of successive approximations, solve the integral equation

$$\varphi(x) = 1 + \int_{0}^{x} (x - t) \varphi(t) dt, \quad \varphi_{o}(x) = 1$$

11 Solve the integro-differential equation.

$$\varphi''(x) + \int_{0}^{x} e^{2(x-t)} \varphi'(t) dt = e^{2x}; \varphi(0) = 0, \varphi'(0) = 1$$

- 12 Solve the integral equation $\varphi(x) = 2 \int_{0}^{1} xt \ \varphi^{3}(t) dt$
- 13 On which curve the functional $\int_{0}^{\frac{\pi}{2}} \left[y'^2 y^2 + 2xy \right] dy$ with y (0) = 0, $y\left(\frac{\pi}{2}\right) = 0$ be extremised.

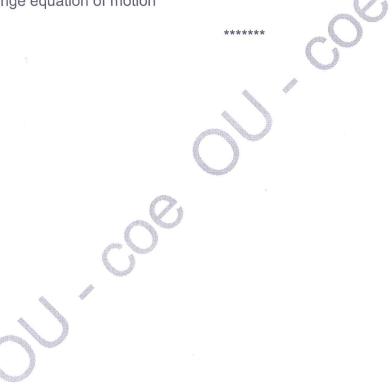
14 Find the extremals of the functional

$$V[y(x), z(x)] = \int_{0}^{\pi/2} [y'^{2} + z'^{2} + 2yz] dx, \quad y(0) = 0, \quad y(\pi/2) = 1, \quad z(0) = 0, \quad z(\pi/2) = -1.$$

15 State the Isoperimetric problem and find the extremals of the problem.

$$V[y(x)] = \int_{0}^{1} (y'^{2} + x^{2}) dx$$
 given that $\int_{0}^{1} y^{2} dx = 2$, $y(0) = 0$, $y(1) = 0$

16 Derive Lagrange equation of motion



FACULTY OF SCIENCE

M.Sc. IV- Semester Examination, October 2020

SUBJECT: Applied Mathematics

Paper - III: Functional Analysis

Time: 2 Hours

Max. Marks: 80

PART - A

Note: Answer any five questions.

(5x7=35 Marks)

- 1. Prove that on a finite dimensional vector space, any two norms or equivalent
- 2. State and prove Translation invariance lemma
- 3. Prove that a linear functional f with domain D (f) in a normed space is continuous if and only if f is bounded
- 4. If in an inner product space $x_n \to x$ and $y_n \to y$, then prove that $\langle x_n, y_n \rangle \to \langle x, y \rangle$.
- 5. Let T be a bounded linear operator on a complex inner product space X and < Tx, x>=0 for all $x \in X$, then prove that T=0.
- 6. Prove that the product of two bounded self adjoint linear operators A and B on a Hilbert space H is self adjoint if and only if AB=BA.
- 7. Let X, Y be normed spaces and S, T ε B (X,Y) Then prove t hat $(i)(S+T)^x = S^x + T^x$ (ii) $(\alpha T)^x = \alpha T^x$ for all scalar α .
- 8. Prove that the normed space X of all polynomials with norm defined by $\|x\| = \max_{j} \|\alpha_{j}\|(\alpha_{0}, \alpha_{1}, \dots \text{ the coeffic} \text{ ients of } x)$ is not complete.

PART - R

Note: Answer any three questions.

(3x15=45 Marks)

- 9. Prove that every finite dimensional subspace Y of a normed space X is complete and closed in X.
- Let $T:D(T\to Y)$ be a bounded linear operator where D (T) lies in a normed space and Y is a Banach space. Then prove that T has an extension $\overline{D(T)}\to Y$ where \overline{T} is a bounded linear operator of norm $\|\overline{T}\| = \|T\|$.
- Prove that the vector space B (X,Y) of all bounded linear operators from a normed space X into a normed space Y is itself a normed space with norm defined by $\|T\| = \frac{S U P}{x \in X} \frac{\|Tx\|}{\|x\|}$.

Also prove that if Y is a Banach space, then B (X,Y) is a Banach space.

- Let X be an inner product space and M a non empty convex subset which is complete in the metric induced by the inner product. Then prove that for every given $x \in X$ there exists a unique $y \in M$ such that $\delta = \frac{\ln f}{y \in M} \|x y\| = \|x y\|$.
- Prove that an ortohonormal set M in a Hilbert space H is total if and only if for all $x \in H$ the Parsevel relation $\sum_{x \in H} |\langle x, y \rangle|^2 = ||x||^2$ holds.
- 14 State and prove Riesz's theorem for sesquilinear form.
- 15 State and prove Hahn-Banach theorem.
- 16 State and prove closed graph theorem.

